STATE ESTIMATION BASED ON UNSCENTED KALMAN FILTER FOR UNDERWATER VEHICLE DYNAMICS

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Abstract - Autonomous underwater vehicles are useful for investigating environmental condition under rivers, lakes, ponds, seas. The wide range of applications of underwater vehicles might motivate researchers to study control system design problems for autonomous underwater vehicles. In general, all the state variables of control systems are not precisely known, because those variables are observed through output sensors and limited parts of them might be only measurable. Hence, automatic control systems must incorporate some type of state estimation. It is needed to establish a state estimation method for underwater vehicle dynamics with restricted measurable state variables. For this purpose, unscented Kalman filter method is applied in this study for estimating the state variables of underwater vehicle dynamics. The objective of this paper is to propose a state estimation method using unscented Kalman filter for underwater vehicle dynamics. The effectiveness of the proposed method is verified by numerical simulations.

Keywords - State Estimation, Observer Systems, Unscented Kalman Filter

I. INTRODUCTION

Over the past few decades, the research interest in underwater vehicles has been increased because they can be used for various tasks in many fields such as rivers, lakes, ponds, seas.An autonomous underwater vehicle (AUV) has become a useful tool for various underwater tasks. For example, AUV can be used for investigating environmental condition under rivers, lakes, ponds, seas. Also, AUV can be used for exploration of oil and gas and for inspection of pipelines and sea cables. In the future, AUV might play an important role to detect victims in flood disaster. The variety of applications of AUVs motivates researchers to study the design problem of the control system for underwater vehicles [1] – [2].

The control problem of AUVs is challenging due to the time-variance and nonlinearity of dynamics. Some classical linear control systems were designed based on a simplified model of underwater vehicle. However, they often resulted in poor performance because the nonlinearity and time-variance of vehicle dynamics is not considered. Therefore, it is desirable to develop a control system with the capability of adapting to the nonlinearity and time-variance of vehicle dynamics. Much efforts have been made in developing nonlinear control methods of AUVs to overcome challenging nonlinear and time-varying problems [3] – [4].

Model predictive control (MPC), also known as receding horizon control [5] -- [10], is a useful control methodology where the control input is determined at each sampling time so as to minimize a given performance index. MPC is a useful control method that enables a control performance to be optimized with considering some constraints on the system state and the control inputs [11] -- [14]. MPC method for the stabilization of nonlinear rotational dynamics of underwater vehicles has been proposed in [15]. However, the control methods proposed in the above papers are inapplicable to systems whose all state variables are not exactly known.

In general, it is usual that the state variables of systems are measured through sensor output. Thus, only limited parts of them can be used for designing control inputs. In other words, it is usual that all the state variables of control systems are not exactly known, because those variables are observed through output sensors and limited parts of them might be only observable. Therefore, automatic control systems must incorporate some type of state estimation. In order to apply the MPC method to the automatic control systems for underwater vehicle dynamics, we need to establish a state estimation method for underwater vehicle dynamics with limited measurable state variables.

The objective of this study is to establish a state estimation method for underwater vehicle dynamics. For this purpose, we introduce an observer system for estimating the state variables of underwater vehicle dynamics. Kalman filter is a well-known optimal estimation method which enables us to minimize estimation errors with taking the process noise and sensor noise into account. The application of the Kalman filter to nonlinear systems has been well investigated in recent decades. The simple approach is to use the Extended Kalman Filter (EKF) [16] which simply linearizes nonlinear models so that the traditional linear Kalman filter can be applied. However, the EKF is only reliable for systems which are almost linear on the time scale of the update intervals. On the other hand, the different method is to use the Unscented Kalman Filter (UKF) [17] that uses a set of appropriately chosen weighted points to parameterize the means and covariances of probability distributions. The state estimator using UKF yields better performance than the one using EKF for control systems with high nonlinearities. Consequently, the objective of this study is to propose a state estimation

method based on the UKF for underwater vehicle dynamics.

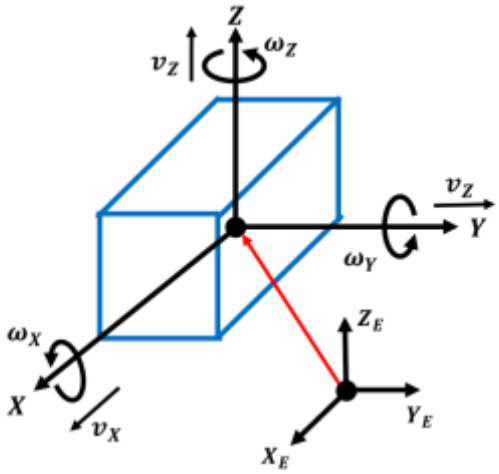


Fig. 1. System model of underwater vehicle

This paper is organized as follows. In Section 2, we define the system model and notations. In Section 3, we consider the state estimation problem of underwater vehicle dynamics. In Section 4, we provide the results of numerical simulations that verify the effectiveness of the proposed method. Finally, some concluding remarks are given in Section 5.

II. NOTATION AND SYSTEM MODEL

In this section, we present the system model for underwater vehicles along with pertinent notations. We start by introducing the system model of a rigid body. Consider a rigid body in an inertial frame of reference, where $v_X(t)$, $v_Y(t)$, $v_Z(t)$ denote the translational velocities of the vehicle along the principal inertial axes, andω_X, ω_Y, and ω_Z denote the angular velocities along a body-fixed reference frame originating from the center of gravity and comprising three principal axes.

2.1. Translational Motion

The translational motion equations of a rigid body, with independent controls aligned along the principal axes, are as follows:

$$m\dot{v}_X(t) = mv_Y(t)\omega_Z(t) - mv_Z(t)\omega_Y(t), \#(1a)$$
$$m\dot{v}_Y(t) = mv_Z(t)\omega_X(t) - mv_X(t)\omega_Z(t), \#(1b)$$
$$m\dot{v}_Z(t) = mv_X(t)\omega_Y(t) - mv_Y(t)\omega_X(t). \#(1c)$$

By including the added moments of inertia J_X , J_Y , andJ_Z, we obtain the following equations.

$$(m + m_X)\dot{v}_X(t) = (m + m_X)v_Y(t)\omega_Z(t) - (m + m_X)v_Z(t)\omega_Y(t),$$
$$(m + m_X)\dot{v}_Y(t) = (m + m_X)v_Z(t)\omega_X(t) - (m + m_X)v_X(t)\omega_Z(t),$$
$$(m + m_X)\dot{v}_Z(t) = (m + m_X)v_X(t)\omega_Y(t) - (m + m_X)v_Y(t)\omega_X(t).$$

2.2. Rotational Motion

The rotational motion equations of a rigid body, with independent controls aligned along the principal axes, are as follows:

$$I_X\dot{\omega}_X(t) = (I_Y + J_Y - I_Z - J_Z)\omega_Y(t)\omega_z(t), \#(2a)$$
$$I_Y\dot{\omega}_Y(t) = (I_Z + J_Z - I_X - J_X)\omega_Z(t)\omega_X(t), \#(2b)$$
$$I_Z\dot{\omega}_Z(t) = (I_X + J_X - I_Y - J_Y)\omega_X(t)\omega_Y(t). \#(2c)$$

By adding the added moments of inertia J_X , J_Y, and J_Z we obtainthe following equations.

$$(I_X + J_X)\dot{\omega}_X(t) = (I_Y + J_Y - I_Z - J_Z)\omega_Y(t)\omega_z(t),$$
$$(I_Y + J_Y)\dot{\omega}_Y(t) = (I_Z + J_Z - I_X - J_X)\omega_Z(t)\omega_X(t),$$
$$(I_Z + J_Z)\dot{\omega}_Z(t) = (I_X + J_X - I_Y - J_Y)\omega_X(t)\omega_Y(t).$$

2.3. Continuous-Time System Model

For simplicity, we introduce the following notations.

$$a_1 := m + m_X, \#(3a)$$
$$a_2 := m + m_Y, \#(3b)$$
$$a_3 := m + m_Z, \#(3c)$$
$$a_4 := \frac{I_Y + J_Y - I_Z - J_Z}{I_X + J_X}, \#(3d)$$
$$a_5 := \frac{I_Z + J_Z - I_X - J_X}{I_Y + J_Y}, \#(3e)$$
$$a_6 := \frac{I_X + J_X - I_Y - J_Y}{I_Z + J_Z}. \#(3f)$$

Furthermore, we define the state vectorx(t) as

$$x(t) := \begin{bmatrix} v_X(t) \\ v_Y(t) \\ v_Z(t) \\ \omega_X(t) \\ \omega_Y(t) \\ \omega_Z(t) \end{bmatrix}. \#(4)$$

Using these notations, the system model can be described by thefollowing state equation:

$$\dot{x}(t) = \bar{f}(x(t)), \#(5)$$
$$\bar{f}(x(t)) := \begin{bmatrix} \frac{a_2}{a_1}x_3x_6 - \frac{a_3}{a_1}x_3x_5 \\ \frac{a_3}{a_2}x_4x_4 - \frac{a_1}{a_3}x_1x_6 \\ \frac{a_1}{a_3}x_5x_5 - \frac{a_2}{a_3}x_2x_4 \end{bmatrix}.$$

The output function y(t) is introduced as:

$$y(t) = Cx(t). \#(6)$$

where C is a coefficient to account for restrictions on sensor allocation.

2.4. Discrete-Time System Model

Let τ denote the discrete-time obtained by the sampling time Δt .Note that the continuous-time

system model (5) can be discretizedinto the following system model.

$$x(\tau+1) = f\big(x(\tau)\big), \#(7a)$$
$$y(\tau) = C\big(x(\tau)\big). \#(7b)$$

The aim of this studyis to propose a state estimation method for the system model described in equation (7).

III. STATE ESTIMATION BASED ONUNSCENTED KALMAN FILTER

In this section, we propose a state estimation method utilizing theUnscented Kalman Filter (UKF) for the system model (7). First, weintroduce the following observer system:

$$\hat{x}(\tau+1) = f\big(\hat{x}(\tau)\big) + z(\tau), \#(8a)$$
$$\hat{y}(\tau) = C\hat{x}(\tau) + w(\tau) \#(8b)$$

where $\hat{x}$ and $\hat{y}$ represent the estimated state and output of x andy, respectively. Additionally, z and w denote the process noise andobservation noise, which may arise from disturbances. In the context of minimum mean-squared error estimation, the optimal stateestimate is determined by the conditional mean.

Let $\hat{x}(i|j)$denote the mean of $\hat{x}(i)$ conditioned on all observations up to and including time j, i.e., $\hat{x}(i|j) = \mathbb{E}[\hat{x}(i)|Y^j]$, where $Y^j := \{\bar{y}(1), \bar{y}(2), \cdots, \bar{y}(j)\}$.It is assumed that the means of $z(\tau)$ and $w(\tau)$ are zero for alltime instances τ.Let $Q^z(\tau)$ and $Q^w(\tau)$ denote the covariances of$z(\tau)$ and $w(\tau)$, respectively.

The UKF [17] initiates by predicting the mean and covariance ofa future state using the process model and weighted sigma points,outlined as follows:

$$\chi^i(\tau+1|\tau) = f\left(\chi^i(\tau)\right), \#(9)$$
$$\hat{x}(\tau+1|\tau) = \sum_{i=0}^{2n} W^i \chi^i(\tau+1|\tau), \#(10)$$
$$Q^{\hat{x}}(\tau+1|\tau) = Q^z(\tau+1) + \sum_{i=0}^{2n} W^i \left(\chi^i(\tau+1|\tau) - \hat{x}(\tau+1|\tau)\right) \#(11) \times \left(\chi^i(\tau+1|\tau) - \hat{x}(\tau+1|\tau)\right)^T. \#$$

WhereW^i and χ^i represent the weight and sigma point, respectively. The definitions of W^i and χ^i can be found in [17]. $\chi^i(\tau+1|\tau)$ can be determined from equation (9). Then, $\hat{x}(\tau+1|\tau)$ and $Q^{\hat{x}}(\tau+1|\tau)$ are computed from equations (10) and (11),respectively.

After redrawing a new set of sigma points $\bar{\chi}^i$ to incorporate theeffect of the additive process noise, the predicted observation iscalculated as:

$$y(t+1|t) = \sum_{i=0}^{2n} W^i C\left(\bar{\chi}^i(\tau+1|\tau)\right). \#(12)$$

Furthermore, the cross covariance P and innovation covarianceR are determined by:

$$P(\tau+1|\tau) = \sum_{i=0}^{2n} W^i \left(\chi^i(\tau+1|\tau) - \hat{x}(\tau+1|\tau)\right) \times \left(C\left(\bar{\chi}^i(\tau+1|\tau)\right) - \hat{y}(\tau+1|\tau)\right)^T \#(13)$$
$$R(\tau+1|\tau) = \sum_{i=0}^{2n} W^i \left(C\left(\chi^i(\tau+1|\tau) - \hat{x}(\tau+1|\tau)\right)\right) \times \left(C\left(\bar{\chi}^i(\tau+1|\tau)\right) - \hat{y}(\tau+1|\tau)\right)^T + Q^w(\tau+1) \#. (14)$$

Consequently, the state estimate at time $\tau+1$ is obtained byupdating the prediction using the linear update rule:

$$K(\tau+1) = P(\tau+1|\tau)R^{-1}(\tau+1|\tau), \#(15a)$$
$$\hat{x}(\tau+1|\tau+1) = \hat{x}(\tau+1|\tau) + K(\tau+1)\big(\bar{y}(\tau+1) - \hat{y}(\tau+1|\tau)\big), \#(15b)$$
$$Q^{\hat{x}}(\tau+1|\tau+1) = Q^{\hat{x}}(\tau+1|\tau) - K(\tau+1)R(\tau+1|\tau)K^T(\tau+1). \#(15c)$$

It is worth noting that the state estimator using UKF yields better performance than the one using EKF for control systems withhigh nonlinearities. EKF simply linearizes nonlinear models so thatthe traditional linear Kalman filter can be applied. On the otherhand, UKF uses a set of appropriately chosen weighted points toparameterize the means and covariances of probability distributions without linearization.

IV. NUMERICAL SIMULATIONS

In this section, we present numerical simulation results to validatethe effectiveness of the proposed method. The initial state and theinitial estimated state are set as follows:

$$x(0) = \left[1, 1, 1, \frac{\pi}{36}, \frac{\pi}{36}, \frac{\pi}{36}\right]^T$$
$$\hat{x}(0) = \left[2, 1, 1, \frac{\pi}{36}, \frac{\pi}{36}, \frac{\pi}{36}\right]^T$$

Table 1 shows the variance of the system noise Q^z, variance ofthe observation noise Q^w, and other parameters used in the numerical simulation. Here, diag denotes a diagonal matrix. The resultsof

numerical simulations using the proposed method, in the casewhere x1 is not observed, are shown below. In Figs. 2--7, the solidblue line represent the time response of the real state x, while thesolid orange line represent the time response of estimated state $\hat{x}$. Itis observed that the estimated state $\hat{x}$ converges to the real state x.Figs. 8--13 show the time responses of the estimation errors. Thesefigures demonstrate the effectiveness of the proposed method.

Simulation parameters	
Q^z	diag[0 ,0 ,0 ,0 ,0]
Q^z	diag[0.1 ,0.1 ,0.1 ,0.175 ,0.175
I_X	0.438
I_Y	0.833
I_Z	0.758
m_X, J_X	34.9
m_Y, J_Y	1.1
m_Z, J_Z	82.5
e_i	$x_i - \hat{x}$

Table 1:Simulation parameters

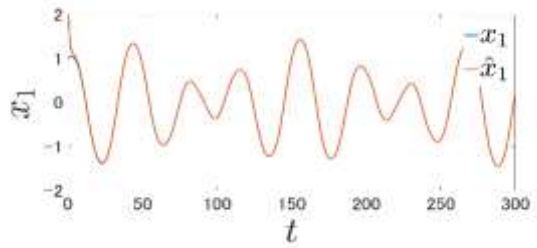


Fig. 2. Time responses of$x_1(t)$ and$\hat{x}_1(t)$

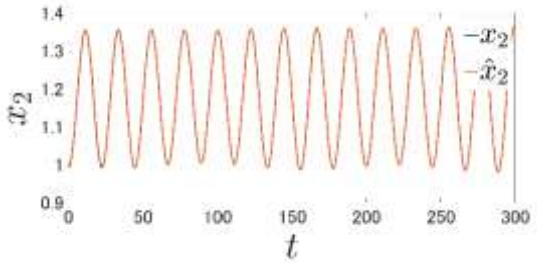


Fig. 3. Time responses of$x_2(t)$ and$\hat{x}_2(t)$

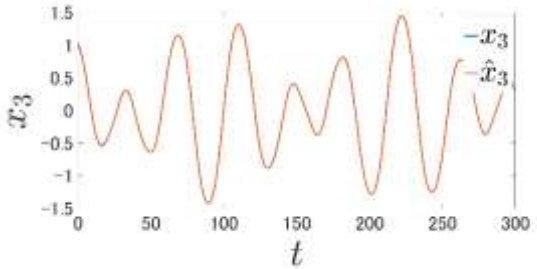


Fig. 4. Time responses of$x_3(t)$ and$\hat{x}_3(t)$

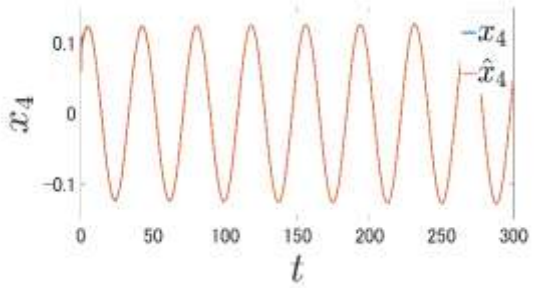


Fig. 5. Time responses of$x_4(t)$ and $\hat{x}_4(t)$

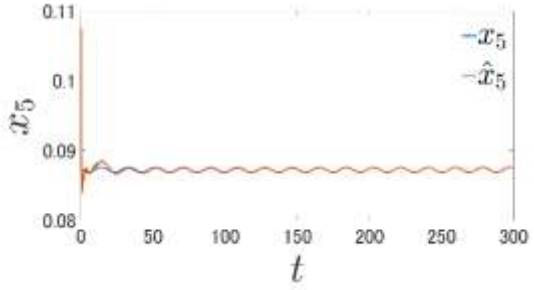


Fig. 6. Time responses of$x_5(t)$ and$\hat{x}_5(t)$

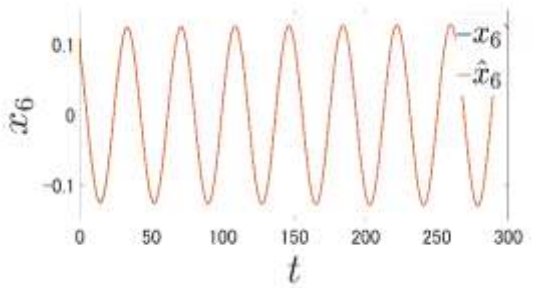


Fig. 7. Time responses of$x_6(t)$ and$\hat{x}_6(t)$

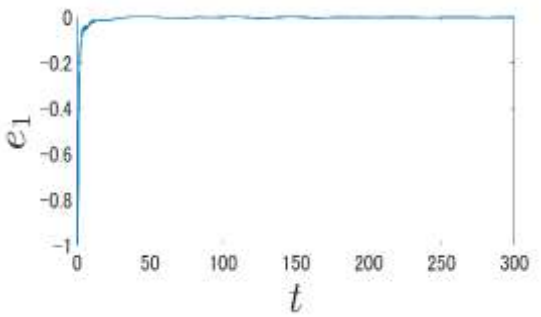


Fig. 8. Time responses ofe_1

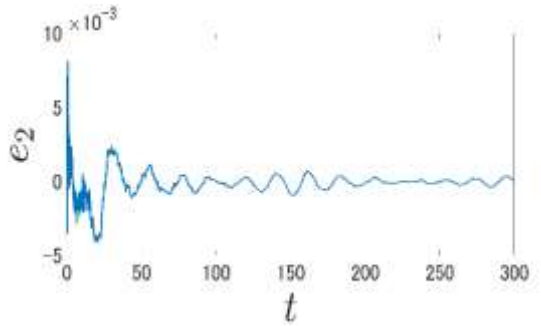


Fig. 9. Time responses ofe_2

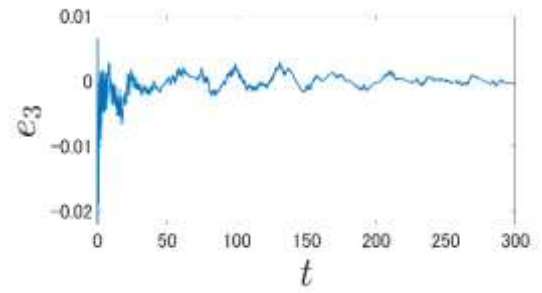


Fig. 10. Time responses ofe_3

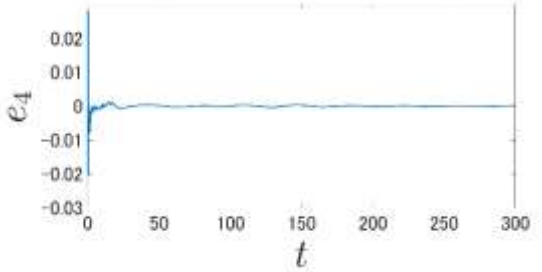


Fig. 11. Time responses ofe_4

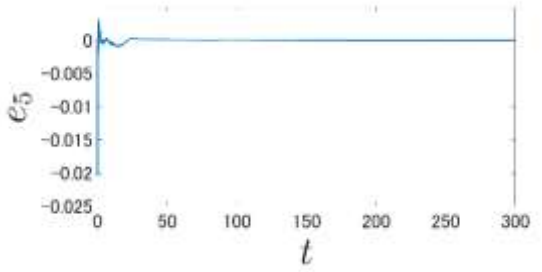


Fig. 12. Time responses ofe_5

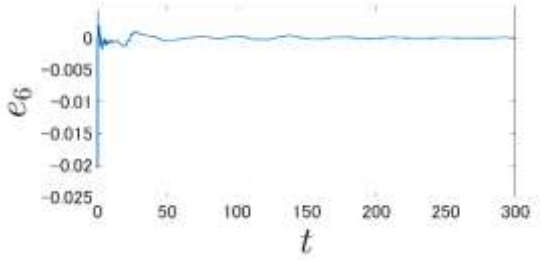


Fig. 13. Time responses ofe_6

V. CONCLUSIONS

The model predictive control method proposed in [15] for underwater vehicle dynamics is inapplicable when all the state variables are not exactly known. In general, the state variables of systems are measured through output sensors, hence, only limited parts of them can be directly known. Thus, it is unrealistic that all the state variables of underwater vehicle system are exactly known for every time. Hence, it should be assumed that the limited state variables can be only known.

To apply the MPC method proposed in [15] to the underwater vehicle control systems, we need to establish a state estimation method for underwater vehicle dynamics with limited measurable state variables. In this study, we established a state estimation method for underwater vehicle dynamics. We proposed a state observer system using the unscented Kalman filter for estimating the state of underwater vehicle system. The effectiveness of the proposed method was verified by numerical simulations. To incorporate the model predictive control with the state estimation method proposed here is a possible future work.

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